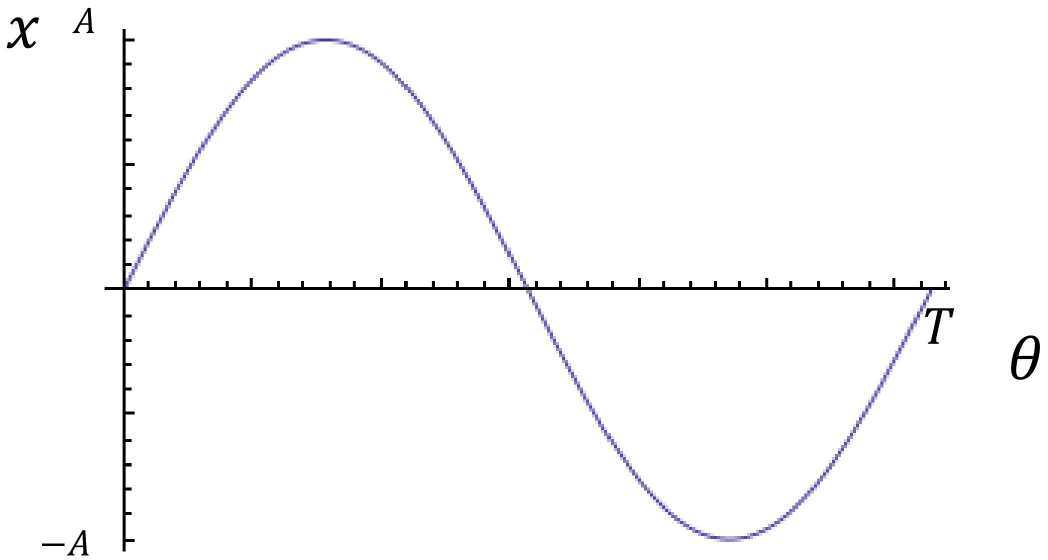
Simple Harmonic Motion

$$Displacement\ x = A\sin\theta$$



$$But,\ when\ \theta = T,\ one\ cycle\ has\ been\ completed$$

$$Let\ \theta = \alpha t$$

$$\alpha T = 2\pi$$

$$\alpha = \frac{2\pi}{T} = \omega$$

$$\boxed{Displacement\ x = A\sin\omega t}$$

$$Displacement\ x = Asin\omega t$$

$$Velocity\ v = \frac{dx}{dt} = A\omega cos\omega t$$

Acceleration $a = \frac{d^2x}{dt^2} = -A\omega^2 sin\omega t$

$$\omega^2 Asin\omega t + -A\omega^2 sin\omega t = 0$$

$$\omega^2 \quad x \quad + \quad a \quad = 0$$

$$x\omega^2 + a = 0$$

$$a = -\omega^2 x$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$a = -(2\pi f)^2 x$$

$$NSL: F = ma$$

$$Restoring\ force:$$

$$\boxed{F = -m(2\pi f)^2 x}$$

Circular Motion

$$Velocity\ v = \frac{x}{t}$$

$$v = \frac{Circumference\ of\ circle}{Time\ Period} = \frac{2\pi r}{T}$$

$$\frac{1}{T} = f\ so,$$

$$\boxed{v = 2\pi rf}$$

$$Angular\ Velocity\ \omega = \frac{d\theta}{dt}$$

$$\omega = \frac{Angle\ of\ full\ cycle}{Time\ Period} = \frac{2\pi}{T}$$

$$\frac{1}{T} = f\ so,$$

$$\boxed{\omega = 2\pi f}$$

Gravitational Fields

A field is a region in space where an object experiences a force.

The gravitational field strength at a point is the gravitational force exerted per unit mass on a small object placed at that point.

$$Gravitaional\ force\ \ g = \frac{F}{m}\ \ Units: N \cdot kg^{-1}\ or\ \ m \cdot s^{2}$$

$$Inverse\ square\ law:\ \ g \propto \frac{1}{r^{2}}$$

$$where\ r = distance\ to\ centre\ of\ mass$$

$$According\ to\ Newton's\ law\ of\ gravitation:$$

$$force \propto product\ of\ masses\ \ \ or\ \ F \propto M \cdot m$$

$$force \propto \frac{1}{distance^{2}}\qquad or\ \ F \propto \frac{1}{r^{2}}$$

$$\therefore F \propto \frac{Mm}{r^{2}}$$

$$\boxed{F = -\frac{GMm}{r^{2}}}$$

$$Where\ G = 6.67 \times 10^{-11} Nm^{2}kg^{-2}$$

$$\textbf{\textit{Kepler's third law}}\ is\ T^2 \propto r^3$$

$$\therefore \frac{T^2}{r^3} = constant$$

$$(this\ is\ the\ same\ for\ all\ planets)$$

$$Force\ of\ attraction = Centripetal\ Force$$

$$F = ma$$

$$\frac{GMm}{r^2} = mr\omega^2$$

$$GM = r^3\omega^2$$

$$GM = r^3\left(\frac{2\pi}{T}\right)^2$$

$$\boxed{T^2 = \left(\frac{4\pi^2}{GM}\right)r^3}$$

Geostationary satellites have an orbital period of 24 hours and are used for telecommunications.

Thermal Physics

$$Absolute\ Temperature\ T \propto E_K$$

Pressure is caused by molecule collisions with the walls of the container

Internal Energy	=	Sum of kinetic and potential energies associated with molecules

$$\boxed{U = \sum_i E_i = \sum E_K + \sum E_P}$$

$$\Delta U = \Delta Q + \Delta W$$

$$Heat\ Capacity\ C = \frac{\Delta Q}{\Delta \theta} = \frac{Energy\ supplied}{Temp\ Change}$$

$$Specific\ Heat\ Capactiy\ c = \frac{\Delta Q}{m\Delta \theta} = \frac{C}{m}$$

$$\boxed{E = mc\Delta\theta}$$

Latent heat is released or absorbed by a substance during a phase transition. Potential energy increases but Kinetic energy is fixed.

$$Specific\ latent\ heat\ L = \frac{Q}{m} \quad Unit: J \cdot kg^{-1}$$

Specific latent heat of a substance is the energy required per kilogram of the substance to change its state without any change in temperature

Gas Pressure

$$Force\ F = rate\ of\ change\ of\ momentum$$

$$F = \frac{mv - mu}{t}$$

$$For\ one\ collision\ with\ the\ wall,$$

$$F = \frac{-mv - mu}{t} = -\frac{2mv}{t}$$

$$For\ a\ population\ of\ N\ molecules$$

$$F_{Total} \approx \frac{Nmv}{t}$$

$$Pressure\ P = \frac{F}{A} = \frac{\frac{Nmv}{t}}{A} = \frac{N}{t} \cdot \frac{mv}{A}$$

$$\frac{N}{t} = collisions\ per\ second$$

therefore pressure is related to the number of collisions per second of the gas molecules with the container

Ideal Gases

An ideal gas is one in which:

1. All collisions between atoms or molecules are perfectly elastic
2. There are no intermolecular attractive forces
3. One can visualise it as a collection of perfectly hard spheres which collide but which otherwise do not interact with each other
4. We assume that the volume of the molecules in the container is negligible compared to the container itself
5. The time of the collision is negligible compared to the time between collisions

In practice, most gases behave ideally only at low pressures at temperatures well above their boiling points.

Gas Law 1 - Boyle's Law

The pressure exerted by a fixed mass of gas is inversely proportional to its volume, provided the temperature of the gas remains constant.

$$p \propto \frac{1}{V}$$

$$pV = k\ (constant)$$

$$p_1V_1 = p_2V_2$$

Gas Law 2 - Charles' Law

The volume of a fixed mass of gas is directly proportional to the temperature, provided the pressure of the gas remains constant.

$$V \propto T$$

$$\frac{V}{T} = k\ (constant)$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Gas Law 3

The pressure of a fixed mass of gas is directly proportional to the temperature, provided the volume of the gas remains constant.

$$P \propto T$$

$$\frac{P}{T} = k\ (constant)$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Combining Laws

$$\frac{pV}{T} = k\ (constant)$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$For\ one\ mole\ of\ gas,$$

$$\frac{pV}{T} = R$$

$$Where\ R = Universal\ gas\ constant$$

$$\boxed{R = 8.31 J \cdot K^{-1} \cdot mol^{-1}}$$

$$For\ n\ moles\ of\ gas,$$

$$\frac{pV}{T} = nR$$

$$\boxed{pV = nRT}$$

One mole of any substance is the amount of that substance which contains the same number of particles as there are in 0.012kg of carbon-12

$$\boxed{Avogadro\ constant\ N_A = 6 \times 10^{23} mol^{-1}}$$

$$Let\ n = number\ of\ moles$$

$$n = \frac{mass\ of\ sample}{molar\ mass} = \frac{m}{M}$$

$$or\ n = \frac{number\ of\ particles}{particles\ per\ mole} = \frac{N}{N_A}$$

$$\frac{Universal\ Gas\ constant}{Avogadro\ constant} = Boltzmann\ constant$$

$$\boxed{k_B = \frac{R}{N_A}}$$

$$k_B = 1.38 \times 10^{-23} J \cdot K^{-1}$$

$$pV = nRT$$

$$\frac{N}{N_A} = n$$

$$pV = \frac{NRT}{N_A}$$

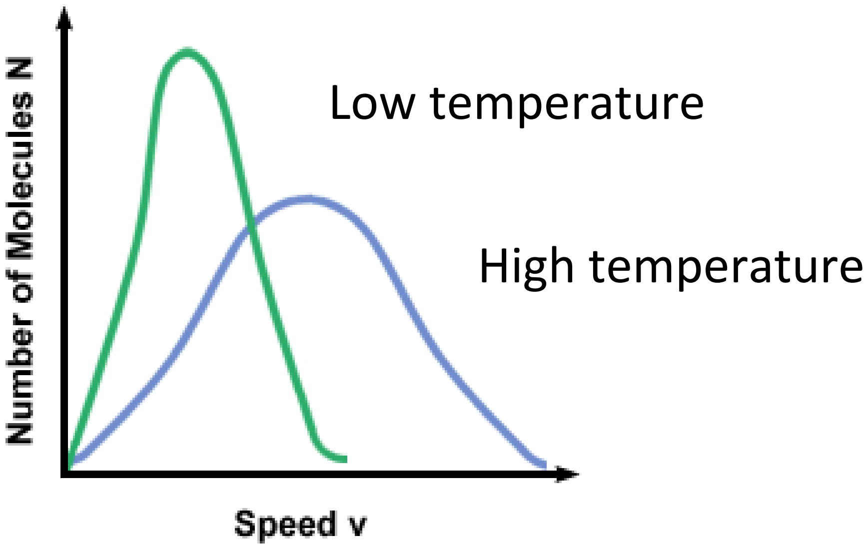
$$\frac{R}{N_A} = k_B$$

$$\boxed{pV = Nk_BT}$$

$$(for\ N\ atoms)$$

Kinetic Theory of Ideal Gas

Normal distribution of molecular speeds



Same area under each graph because population does not change

$$Mean\ speed = \bar{c}$$

$$Average\ E_K = \frac{1}{2}m\bar{c}^2$$

$$E_K \propto T$$

$$\boxed{\frac{1}{2}m\bar{c}^2 = \frac{3}{2}k_BT}$$